Möhle, Martin (2014) On hitting probabilities of beta coalescents and absorption times of coalescents that come down from infinity. *ALEA Lat. Am. J. Probab. Math. Stat.* **11**, 141–159.

This file contains corrections concerning the article mentioned above. Moreover, two additional remarks are provided, which could be helpful for the reader.

Corrections.

1. Page 153, Case 3: The part '(see remark after Lemma 4.4)' should be removed, since there is no such remark. The statement that there exists a constant C such that $|b_n(k)-1|\le Ck/n$ is not correct, since, for $k=n-1$,

$$b_n(n-1) \;=\; \frac{\Gamma(n)\Gamma(\alpha)}{\Gamma(n+\alpha-1)} \;\sim\; \Gamma(\alpha)n^{1-\alpha} \;\to\; \infty, \qquad n\to\infty.$$

Therefore, the proof concerning Case 3 has to be modified from page 153, line 12 on as follows. Split the sum $S:=\sum_{k=m-1}^{n-1} b_n(k)a_{k-m+1}/k$ into two parts $S=S_1+S_2$, where

$$S_1 \;:=\; \sum_{k=m-1}^{\lfloor n/2\rfloor} b_n(k)\frac{a_{k-m+1}}{k} \quad\text{and}\quad S_2 \;:=\; \sum_{k=\lfloor n/2\rfloor+1}^{n-1} b_n(k)\frac{a_{k-m+1}}{k}.$$

Since $\alpha\in(0,1)$ it follows that $b_n(k)=\prod_{j=1}^k (n-j)/(n-j+\alpha-1)$ is non-decreasing in $k\in\{1,\ldots,n-1\}$ and, hence,

$$|S_2| \;\le\; \frac{b_n(n-1)}{n/2}\sum_{k=m}^{\infty}|a_{k-m+1}| \;=\; 2\alpha\frac{b_n(n-1)}{n} \;\sim\; 2\alpha\frac{\Gamma(\alpha)}{n^\alpha} \;\to\; 0, \qquad n\to\infty.$$

Therefore, $S_2\to 0$ as $n\to\infty$. It remains to consider S_1. For $k\le\lfloor n/2\rfloor$ we have

$$1 \;\le\; b_n(k) \;\le\; b_n(\lfloor n/2\rfloor) \;=\; \frac{\Gamma(n)\Gamma(n-\lfloor n/2\rfloor+\alpha-1)}{\Gamma(n-\lfloor n/2\rfloor)\Gamma(n+\alpha-1)} \;\sim\; \frac{(n-\lfloor n/2\rfloor)^{\alpha-1}}{n^{\alpha-1}} \;\sim\; 2^{1-\alpha}$$

as $n\to\infty$. Thus it is allowed to apply dominated convergence to the sum S_1 (interpreted as an integral with respect to the counting measure on $\{m-1,m,\ldots\}$), which yields

$$S_1 \;=\; \sum_{k=m-1}^{\lfloor n/2\rfloor} b_n(k)\frac{a_{k-m+1}}{k} \;\to\; \sum_{k=m-1}^{\infty}\frac{a_{k-m+1}}{k} \;=\; \int_0^1 \frac{t^{m-1}}{L_\alpha(t)}\,\mathrm{d}t, \qquad n\to\infty.$$

2. Page 157: In the last displayed line $\mathrm{E}(\tau)$ should be replaced by $\mathbb{E}(\tau)$.

Remark 1. Theorem 2.1 on p. 143 provides the main formula (2.2) for the hitting probability $h(n,m)$ of the block counting process of the $\beta(2-\alpha,\alpha)$-coalescent with parameter $\alpha\in(0,2)$. We verify below that (2.2) reduces for $\alpha=1$ (Bolthausen–Sznitman coalescent) to the formula (11) in [1].

Proof. Clearly, for $\alpha = 1$, (2.2) reduces to

$$h(n,m) \;=\; (m-1)\sum_{k=m-1}^{n-1}[z^k]\int_0^z \frac{t^{m-1}}{L_1(t)}\,\mathrm{d}t.$$

By (2.3) and (4.11),

$$[z^k]\int_0^z \frac{t^{m-1}}{L_1(t)}\,\mathrm{d}t \;=\; \frac{1}{k}\sum_{j=1}^{k-m+1}(-1)^j \sum_{\substack{n_1,\ldots,n_j\in\mathbb{N}\\ n_1+\cdots+n_j=k-m+1}} \frac{1}{(n_1+1)\cdots(n_j+1)} \;=\; \frac{a_{k-m+1}}{k},$$

where $a_0, a_1, \ldots$ are the coefficients in the Taylor expansion $z/L_1(z) = \sum_{j=0}^\infty a_j z^j$. Thus,

$$h(n,m) \;=\; (m-1)\sum_{k=m-1}^{n-1}\frac{a_{k-m+1}}{k} \;=\; (m-1)\sum_{j=0}^{n-m}\frac{a_j}{j+m-1},$$

which is Eq. (11) of [1]. Note that in [1] the alternative representation $a_j = (-1)^j/j!\int_0^1 (x)_j\,\mathrm{d}x$ for the coefficient a_j is used (see [1, Lemma 3.1]).

Remark 2. On p. 155 at the beginning of the proof of Theorem 3.3 it is stated that τ_∞ almost surely coincides with τ. We verify below the slightly stronger result that $\tau_\infty(\omega) = \tau(\omega)$ for all $\omega \in \Omega$.

Proof. Fix $\omega \in \Omega$. Clearly, $\{t > 0 : N_t(\omega) = 1\} \subseteq \{t > 0 : N_t^{(n)}(\omega) = 1\}$ for all $n \in \mathbb{N}$ and, hence, $\tau_n(\omega) := \inf\{t > 0 : N_t^{(n)}(\omega) = 1\} \le \inf\{t > 0 : N_t(\omega) = 1\} =: \tau$ for all $n \in \mathbb{N}$. Taking the limit $n \to \infty$ it follows that $\tau_\infty(\omega) \le \tau(\omega)$.
Assume now that $\tau_\infty(\omega) < \tau(\omega)$. Then there exists $t = t(\omega) \in (0,\infty)$ such that $\tau_\infty(\omega) < t < \tau(\omega)$. Since $\tau_n(\omega) \le \tau_\infty(\omega)$ it follows that $\tau_n(\omega) < t$ and hence $N_t^{(n)}(\omega) = 1$ for all $n \in \mathbb{N}$. But this implies that $N_t(\omega) = 1$ and hence $\tau(\omega) \le t$ in contradiction to $t < \tau(\omega)$. Thus, the assumption is wrong and we have $\tau_\infty(\omega) = \tau(\omega)$.

References

[1] Möhle, M. (2014) Asymptotic hitting probabilities for the Bolthausen–Sznitman coalescent. *J. Appl. Probab.* **51A**, 87-97. MR3317352